C4 Jan 2007

1) $\boxed{3}$ $\frac{(x+6)(x-4)}{x(x-4)} = \frac{x+6}{x}$ (2) (1)

2) $\int x \ln x \, dx = \int \ln x \; x \, dx$

$u = \ln x \quad \frac{dv}{dx} = x$ (1)

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^2}{2}$

$I = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}\,dx$ (1)

$\boxed{5}$ $= \frac{x^2}{2}\ln x - \int \frac{x}{2}$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4}$ (1)

$A(2) = 2\ln 2 - 1$ (1)

$A(1) = -\frac{1}{4}$

$\text{Ans} = 2\ln 2 - \frac{3}{4}$ (1)

3) $\vec{AB} = b's - a's = 11i - 2j - 6k$ (1)

$|AB| = \sqrt{11^2 + 2^2 + 6^2}$

$= \sqrt{161} = 12.7$ (2)

4) find $\angle$ OAB do AC . AB

$\vec{AC} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} 11 \\ -2 \\ -6 \end{pmatrix}$ (1)

$AC . AB = -44 + 6 - 12 = -50$

$= \sqrt{29}\sqrt{161}\cos\theta$ (1)

$\boxed{6}$ $\cos\theta = \frac{-50}{\sqrt{29} + \sqrt{161}}$ (1)

$\theta = 137^\circ$ so $\angle = 43^\circ$

4) $u = 2x - 5 \quad \frac{du}{dx} = 2$

$I = \int (4x-8)(2x-5)^7 = \int (4x-8)u^7 \times \frac{du}{2}$ (1)

but if $u = 2x - 5$

$2u = 4x - 10$

so $2u + 2 = 4x - 8$

$I = \int (2u+2)u^7 \times \frac{du}{2}$ (1)

$\boxed{5}$ $= \int u^8 + u^7 \, du = \frac{u^9}{9} + \frac{u^8}{8}$ (1)

if $x = 3 \quad u = 1$

$x = \frac{5}{2} \quad u = 0$ (1)

$I(1) = \frac{1}{9} + \frac{1}{8} \quad I(0) = 0$

$I = \frac{17}{72}$ (1)

5) $(1-3x)^{-\frac{1}{3}} \quad n = -\frac{1}{3} \quad x = -3x$

$= 1 + x + 2x^2 + \frac{14}{3}x^3$ (4)

replace x by $x + x^3$

$= 1 + (x + x^3) + 2(x^2 + 2x^4 + x^6)$

$\boxed{7}$ $+ \frac{14}{3}(x^3 + 3x^5 \ldots)$ (3)

coefft $x^3 = 1 + \frac{14}{3} = \frac{17}{3}$

6) $\frac{2x+1}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2}$

$2x + 1 = A(x-3) + B$ (1)

$x = 3 \quad 7 = B$

$x = 0 \quad 1 = -3A + 7 \quad A = 2$

$f(x) = \frac{2}{x-3} + \frac{7}{(x-3)^2}$ (2)

ii) $\int \frac{2}{x-3} + \frac{7}{(x-3)^2} \, dx$

$\int \frac{2}{x-3} \, dx = 2\ln(x-3)$ 1

$\int 7(x-3)^{-2} = 7\frac{(x-3)^{-1}}{-1} = -\frac{7}{x-3}$ 1

$\boxed{7}$

$I = 2\ln(x-3) - \frac{7}{x-3}$

$I_{10} = 2\ln 7 - 1$ 1

$I_4 = 2\ln 1 - 7 = -7$

$I = 2\ln 7 - 1 - -7 = 2\ln 7 + 6$ 1

$a = 6 \quad b = 2 \quad c = 7$

7) $2x^2 + xy + y^2 = 14$

$4x\,dx + x\,dy + y\,dx + 2y\,dy = 0$

$\div dx \quad 4x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ 2

$\frac{dy}{dx}(x+2y) = -4x - y$

$\frac{dy}{dx} = \frac{-4x-y}{x+2y}$ 1

for TP $\quad -4x - y = 0$ 1

$y = -4x$ 1

subst in original

$2x^2 + x(-4x) + 16x^2 = 14$ 1

$14x^2 = 14$

$x = \pm 1$ 1

$y = \mp 4$ 1

TP's $(1,-4) \quad (-1,4)$

$\boxed{8}$

8) i) $x = 2t^2 \quad \frac{dx}{dt} = 4t$

$y = 4t \quad \frac{dy}{dt} = 4$

$\frac{dy}{dx} = 4 \times \frac{1}{4t} = \frac{1}{t}$ 1

grad normal $= -t$

but at $P(2p^2, 4p)$ $t = p$

so grad normal $= -p$ 1

ii) grad chord $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4q - 4p}{2q^2 - 2p^2} = \frac{\cancel{4}^{2}\cancel{(q-p)}}{2(q+p)\cancel{(q-p)}}$ 1

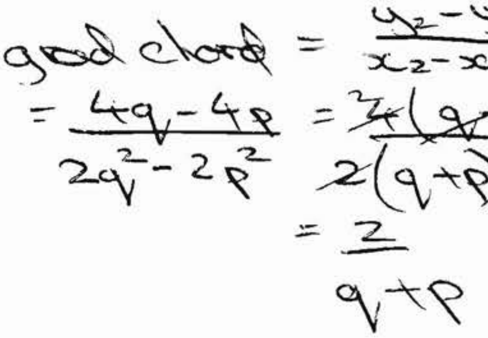
$= \frac{2}{q+p}$ 1

iii) if chord is the normal then the grads are equal

$-p = \frac{2}{q+p}$ 1 $\Rightarrow -pq - p^2 = 2$

$p^2 + pq + 2 = 0$ 1

iv)



$R(8,8) \Rightarrow p = 2 \quad (2p^2, 4p)$

so $2^2 + 2q + 2 = 0$

$q = -3$ 1

$\boxed{10}$

if $q = -3$ at S 1

$(-3)^2 - 3p + 2 = 0 \quad p = \frac{11}{3}$

at T $p = \frac{11}{3}$ ie $\left(2\times\left(\frac{11}{3}\right)^2, 4\times\frac{11}{3}\right)$

$T\left(\frac{242}{9}, \frac{44}{3}\right)$ 1

9) $\int \sec^2 y \, dy = \int 2\cos^2 2x \, dx$ (1) $\quad \cos 2x = 2\cos^2 x - 1$

$\cos 4x = 2\cos^2 2x - 1$

$\int \sec^2 y \, dy = \int \cos 4x + 1$ (2)

(1) (1)

$\boxed{10}$ $\tan y = \frac{1}{4}\sin 4x + x + c$

ii) $x = 0 \quad y = \frac{\pi}{4} \quad 1 = c$ (1) $\Rightarrow \tan y = \frac{1}{4}\sin 4x + x + 1$

if $x = \frac{\pi}{6} \quad \tan y = \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{\pi}{6} + 1 = 1.74$ (1)

$y = 1.049$ (1) work in RADS

10) $P\begin{pmatrix}5\\2\\-9\end{pmatrix} \quad Q\begin{pmatrix}4\\4\\-6\end{pmatrix}$ line PQ $r = \begin{pmatrix}5\\2\\-9\end{pmatrix} + s\begin{pmatrix}-1\\2\\3\end{pmatrix}$ (2) ie $p + t(q - p)$ or other way

ii) line OT $r = \begin{pmatrix}0\\0\\0\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix}$ (1)

dir PQ $= \begin{pmatrix}-1\\2\\3\end{pmatrix}$ (1) dir OT $= \begin{pmatrix}1\\2\\-1\end{pmatrix}$ (1) PQ.OT $= -1 + 4 - 3$

$= 0$ (1)

hence $\perp$

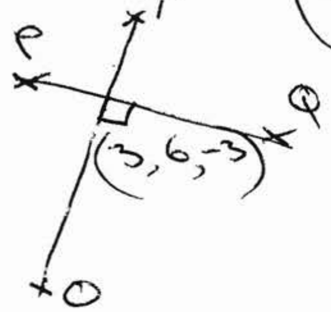
iii) at intersection $\begin{pmatrix}5\\2\\-9\end{pmatrix} + s\begin{pmatrix}-1\\2\\3\end{pmatrix} = t\begin{pmatrix}1\\2\\-1\end{pmatrix}$

x's $5 - s = t$ (1) y's $2 + 2s = 2t$ (1)

solving $s = 2 \quad t = 3$

intersect at $\begin{pmatrix}3\\6\\-3\end{pmatrix}$ using $t = 3$ in OT eqn (1)

iv)



$\perp$ distance $= \sqrt{3^2 + 6^2 + (-3)^2}$

$= \sqrt{54}$

$= 3\sqrt{6}$ (2)

$\boxed{11}$